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Optimal design and operation of SMB bioreactor for sucrose inversion

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Abstract

A comprehensive optimization study was carried out to evaluate the performance of a simulated moving bed reactor (SMBR) system for an industrially important biochemical reaction-separation problem, the inversion of Sucrose and the in situ separation of the products, glucose and fructose. Two modifications of SMBR are studied, one in which non-synchronous switching is used to vary the number of columns in different sections within a global switching period (the so-called Varicol® process), while in the other the concept of distributed feed is studied in which the feed flow was distributed over the global switching period. Multi-objective optimization study is performed as it results in more meaningful solutions although the optimization for such complex processes is complicated owing to the complex interplay of relatively large number of decision variables, including continuous variables like switching time, flow rates and column length and discrete variables like number and distribution of columns in different sections. A state-of-the-art non-traditional but more versatile optimization technique based on genetic algorithm, non-dominated sorting genetic algorithm with jumping genes (NSGA-II-JG) was used. The optimization results in significant improvement for SMBR as well as its modification, Varicol and distributed feed systems, in terms of increasing productivity using less desorbent for both existing as well as at design stage.

Keywords: Simulated moving bed; Varicol®; Multi-objective optimization; Pareto set; Sucrose inversion; Fructose syrup; Genetic algorithm

1. Introduction

Chromatography is a separation technique widely implemented not only at the small analytical scale, but also at preparative and at large industrial scale. In this framework, the simulated moving bed (SMB) technology [1] is an established technique for continuous chromatographic separations of various mixtures including hydrocarbon isomers, sugars and different fine chemicals, such as natural products, pharmaceuticals, aromatics and enantiomers. The SMB technique implies a simulated countercurrent contact between the mobile fluid phase and the stationary adsorbent phase. There are many advantages [2,3] of the SMB technology compared to the classical preparative chromatography, namely, overcoming problems associated with solid handling (movement, attrition, channeling, etc.), efficient utilization of adsorbent (and catalyst), continuous mode of operation, lowering solvent consumption (with up to 90% saving compared to classical preparative chromatography), reducing downtime (as separation and regeneration takes place concurrently), possibility of scaling up to a large-scale production unit, etc.

Chromatographic reactors are systems that are used to convert one or more components (partially or totally) and simultaneously separate one or more of the products that are being formed. Reaction occurs either in the mobile or within the pores of the stationary phase. In the latter case, the catalyst is supported or immobilized on the solid adsorbent, which promotes the separation of the reaction products. In situ separation of the products facilitates the reversible reaction to completion beyond thermodynamic equilibrium and at the same time obtaining products of high purity. Chromatographic reactor based on SMB technology, namely simulated moving bed reactor (SMBR), provides economic benefit for equilibrium limited reversible reactions, such as many hydrogenation, isomerization and esterification reactions [4–6]. They are also unique in applications where the removal of

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Nomenclature

Bi_m	mass Biot number, –
C	fluid phase concentration, g/l
D_{ax}	axial diffusivity, m^2/s
K	adsorption equilibrium constant
K_e	linear adsorption constant of the enzyme
K_{mm}	Michaelis–Menten constant
k_h	mass-transport coefficient (LDF), s^{-1}
k_f	film mass-transport coefficient, m/s
k_p	mass-transport coefficient in the pores, s^{-1}
k_r	reaction rate constant, s^{-1}
k_μ	mass-transport coefficient in micro-particles, s^{-1}
L_{col}	column length, cm
N	adsorbed phase saturation concentration
Pe	Peclet number, –
PR	productivity, kg/m^3 solid/h
Pur	purity
$\bar{q}$	adsorbed concentration averaged over the particle volume, mol/l
$\bar{q}^*$	concentration at the particle surface in equilibrium with bulk fluid concentration, mol/l
Q_{fi}	feed flow rate in subinterval 'i', ml/min
Q_i	flow rate in section 'i', ml/min
R_p	radius of the pore, cm
t_s	switching period, min
U_F	fluid interstitial velocity

Greek letters

α	number of mass-transfer units for a homogeneous adsorbent particle (LDF)
ε	bed porosity
ε_p	particle porosity
ν	solid to fluid volume ratio
ζ	pseudo solid velocity, cm/min
χ	column configuration

Superscripts and subscripts

D	eluent (desorbent)
Enz	enzyme
E	extract
F	fructose
f	feed
G	glucose
Suc	sucrose
R	raffinate

inhibitors, acceptor products or poisons improves the overall reaction yield. This is particularly true in bioreactors catalyzed by microorganisms since when operated with product concentration within a certain physiological range they are very efficient [7]. Besides, a build-up of product concentration may lead to the inhibition of the process concerned and thus, limit the productivity.

The application of chromatographic reactors in the biochemical field was initiated in the early 1980s by Barker and Ganetsos [8]. Other ingenious arrangements that implement the principle of continuous chromatographic reaction-separation principle have been reported [9–13]. The inversion of sucrose, the isomerization of glucose to fructose and the biosynthesis of dextran from sucrose were the test reactions in all these works. Financial and operational benefits were achieved through such process integration.

Industrial processes aim at maximizing their production capacities while simultaneously improving the product quality and reducing operating costs. Usually, there exists a trade-off between these requirements. This is particularly true in the production of high fructose corn syrup (HFCS) using SMB (or Varicol®) systems where high productivity at reduced eluent consumption is the most important issue. Thus, in this case the design and operation of SMB (and Varicol®) systems need to be optimized using multiple objective functions and constraints. In multi-objective optimization, a solution that is the best (global optimum) with respect to all objectives may not exist, as these objectives are often conflicting [14]. Instead, there could exist an entire set of optimal solutions that are equally good, known as the Pareto-optimal solutions. A Pareto set is defined such that when one moves from one point to another, at least one objective function improves and at least one other worsens. So, no point on this curve is superior to any other solutions. The choice of a solution over the other solutions requires additional knowledge of the problem, and often this knowledge is intuitive and non-quantifiable. The Pareto-set narrows down the choices and helps to guide a decision-maker in selecting a desired operating point (called the preferred solution) from among the (restricted) set of Pareto-optimal points, rather than from a much larger number of possibilities [15]. It should be noted that in our study, we did not consider profit (or cost) as objective function as they are site and time specific and hence, optimal solution will be meaningless as cost of raw materials and products varies from country to country and at with time.

There are a number of publications focusing on the design of non-reactive SMB [16–19] as well as reactive SMB [20–23]. Comparatively, very few studies have been reported on the optimization of SMB systems [24,25]. However, most of the investigations are based on single objective optimization, incorporating several objectives with some weight factors except few recently reported works [24,25]. Single objective function optimization approach is not efficient and also has the drawback of possibly losing certain optimal solutions when the non-convexity of the objective function gives rise to a duality gap [14,15], something that is very difficult to ensure for more complex, real-life problems.

In this article, a systematic optimization study on the applicability of SMBR (and reactive Varicol®) for the inversion of sucrose in the presence of the enzyme, invertase, and in situ separation of the products, viz., glucose and fructose, is

discussed at length. An existing model that can predict published pilot-scale experimental results of the system is used in the optimization. The model was verified first with published experimental results [5], and thereafter, few multi-objective optimization studies were carried out to obtain the Pareto-optimal solutions to provide a clear distinction between the performances of the SMB and the Varicol® process. The optimal operating parameters (such as the feed and desorbent flow rates, the flow rates in the different sections, the switching time and amount of catalyst) and geometric parameters (such as number and length of columns, and its switching sequence) are determined. The state-of-the-art evolutionary optimization technique, elitist non-dominated sorting genetic algorithm with jumping genes, NSGA-II-JG [26] has been used in obtaining the Pareto-optimal solutions.

2. Enzymatic inversion of sucrose in SMBR

In the last couple of decades a substantial amount of research has been conducted for the separation of glucose and fructose in SMB systems [3,11,27,28]. Both glucose and fructose have linear isotherms over a wide concentration range and hence are an excellent experimental mixture used in developing analytical and numerical simulations for the performance prediction and design purposes. Recently, inversion of sucrose in presence of invertase has also been studied for the design of bioreactive SMB systems [5]. The separation is performed using ion-exchange resins with hot water as the desorbent. The preferred implementation consists of using polystyrene cation-exchange resins in the calcium form in which the fructose forms a complex with the calcium ions and is retarded, while the glucose and the other oligosaccharides are eluted with the eluent.

Invert sugar syrup, an equimolecular mixture of glucose and fructose, is a valuable sweetener and is required by the food and pharmaceutical industries. Fructose is 1.3 times sweeter than sucrose and about 1.7 times sweeter than glucose. Furthermore, it has functionally more desirable properties like low carcinogenity, high osmotic pressure, high solubility, source of instant energy and prevents crystallization of sugar in food products. Hence, high fructose syrups are in great demand as food and soft drink sweeteners. There are different methods of inversion among which acid inversion and bio-inversion are more popular. Bio-inversion though expensive, is a better alternative, as it does not produce any polymerized byproducts as seen in case of acid inversion. Hydrolysis of sucrose by the enzyme invertase leads to the production of invert sugar syrup. Yeast cells are commonly used as a source of invertase. The inversion of sucrose takes place as follows:

$$\text{Sucrose} \xrightarrow{\text{invertase}} \text{glucose} + \text{fructose} \quad (1)$$

As seen in Eq. (1), inversion of sucrose is an irreversible reaction, and thus, the reaction rate is not influenced by the product accumulation. However, it has been shown that even for irreversible reactions, the use of a simulated moving bed reactor (SMBR) increases conversion and product purity as compared to the performance of an equivalent chromatographic reactor-separator in batch mode [13,29]. Barker et al. [10] have also shown that simultaneous inversion and product separation makes it possible to overcome problems associated with substrate inhibition. This further justifies the use of SMBR systems in the industrial production of high fructose sugar solutions from the cane sugar (sucrose). Furthermore, use of SMB technology, helps in reducing total amount of water in the product stream, as the major expense is the cost of evaporating water (sometimes in combination with reverse osmosis) from the product stream. Water removal has a major effect on design, investment and operation costs of these sugar industries.

SMB is a practical implementation of the true moving bed (TMB) process in which the countercurrent movement between the mobile phase and the stationary phase in TMB is simulated by moving the input/output ports periodically and in tandem along a series of fixed columns in the direction of the mobile phase flow, while holding the bed stationary [29,30]. Hence, periodic discrete steps in the SMB replace the continuous motion of the fluid and solid in the TMB. In the Varicol® operation [31,32], a recent modification of the conventional SMB process, a non-synchronous shift of the inlet and outlet ports is employed within a (global) switching period although the switching period is decided a priori and kept constant. During one global switching period, there are different column configurations for sub-time intervals due to local switching. Given the total number of columns employed in a Varicol® process, the number of columns in each zone varies with time within a global switching period. As a result, Varicol® process can have several column configurations, which endow more flexibility compared to conventional SMB process. SMB process can be regarded as the most rigid and a special case of more flexible Varicol® process without adding any additional fixed cost.

A schematic representation of a SMB system is illustrated in Fig. 1 that consists of a number of columns of uniform cross-section connected in a circular array, each of length L_{col} and packed with the ion exchange resin adsorbent. The two incoming streams (the feed, F and the desorbent, D) and the two outgoing streams (the raffinate, R and the extract, E) divide the system into four sections namely P, Q, R and S, each of which comprising p, q, r and s columns, respectively. The flow rates in the section P, Q, R and S were designated as Q_P, Q_Q, Q_R and Q_S, respectively while those of the feed, raffinate, desorbent and extract were designated Q_F, Q_R, Q_D and Q_E, respectively. However, only four of the above eight flow rates are independent, as the remaining four are determined from the mass balance at points A–D (see Fig. 1). In the work of Azevedo and Rodrigues [5], the SMBR set-up had 5, 2, 3 and 2 columns in sections P, Q, R and S, respectively as shown in Fig. 1. The feed was a diluted sucrose solution (80 g/l), since the Michaelis–Menten equation has been shown to apply at

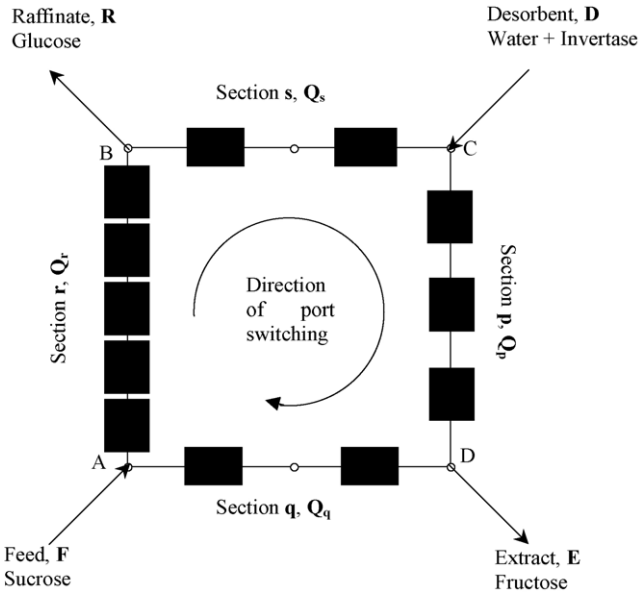

Fig. 1. Schematic diagram of a 12-column SMBR pilot system. The inlets and outlets divide the entire SMBR system into four sections, p, q, r and s with respectively, 3, 2, 5 and 2 columns. Q_p, Q_q, Q_r and Q_s denote the flow rates in each section, respectively.

this concentration. Besides, it was reported that the available SMBR set-up could not withstand the high pressure-drops that would result from the high viscosity of concentrated sucrose syrups due to the small diameter of the tubing (1/16") connecting the columns. The enzyme invertase was fed to the SMBR diluted in the desorbent (warm water). Its maximum activity was observed at 328 K and at pH 4.5. Therefore, the eluent consisted of a pH 4.5 buffer prepared from acetic acid (0.28%, v/v) and calcium acetate (0.5%, w/v).

The internal fluid flow rates and the simulated solid flow rate must be selected appropriately to achieve the desired separation performance. By suitably advancing the inlet and outlet ports, column by column, in the direction of the fluid flow at a pre-set switching time, t_s, the countercurrent movement of the solids can be mimicked. This switching time is the key parameter, which defines the hypothetical solid-phase velocity. However, countercurrent separation of the components could be achieved only by appropriately specifying the internal flow rates in the columns and the switching time. Petroulas et al. [33] defined for true countercurrent moving bed chromatographic reactor (CMCR) a parameter, σ_i, called relative carrying capacity of the solid relative to the fluid stream for any component i as:

$$\sigma_i = \frac{1-\varepsilon}{\varepsilon} NK_i \frac{u_s}{u_g} = \delta_i \frac{u_s}{u_g} \tag{2}$$

where u_s and u_g are respectively solid-phase and fluid-phase velocity. They showed that to achieve countercurrent separation between the two components, one must set σ greater than 1 for one and less than 1 for the other. Later, Fish et al. [34] verified the above fact experimentally. Fish et al. [34] also defined V_i, the net velocity at which component i travels (or the concentration front moves) within the column, which for linear isotherm is given by:

$$V_i = \frac{u_g \lfloor 1-\sigma_i \rfloor}{1+\delta_i} \tag{3}$$

Therefore, when $\sigma_i < 1$, $V_i > 0$ (species move with the fluid phase), and when $\sigma_i > 1$, $V_i < 0$ (species move with the solid phase). When $\sigma = 0$, it represents fixed bed. Ray et al. [4] re-defined the above parameter, σ, by replacing the solid-phase velocity, u_s, in CMCR by a hypothetical solid-phase velocity, ζ, defined as $\zeta = L_{col}/t_s$. They found, both theoretically [29] and experimentally [30], that simulation of the countercurrent movement between two components can be achieved when re-defined σs were set such that it is greater than 1 for one and less than 1 for the other component. Hence, in the present study if we set σ properly, the more strongly adsorbed component (fructose) will move with the solid (resin) stream, and can be collected at the extract port (point D in Fig. 1), while at the same time the less strongly adsorbed component (glucose) will travel with the fluid stream, and can be collected at the raffinate port (point B in Fig. 1). It is worth noting that in SMB the switching time and column configuration (the number of columns in each section) is decided a priori and is maintained constant during the entire operation.

Unlike SMBR as shown in Fig. 2a for an eight-column system, the reactive Varicol® [35] is based on a non-simultaneous and unequal shift of the inlet and outlet ports. Fig. 2b illustrates and compares the principles of operation of an eight-column four sub-time interval Varicol® system with an SMBR for one switching interval. The switching time t_s, is a key parameter in the Varicol® process also, although the relationship is not straightforward. The solid velocity is not a constant value since the zone length and the number of columns in each zone varies during the switching period. Within a global switching period, t_s, for a four-subinterval Varicol® system, the column configuration may change for each quarter of t_s. For example, the column configuration for a typical sequence in a given cycle corresponding to Fig. 2b changes from 2/1/2/3 (0–1/4t_s) to 3/1/1/3 (1/4t_s–1/2t_s) by shifting both the feed and the extract port by exactly one column in the backward direction, then to 3/1/2/2 (1/2t_s–3/4t_s) by shifting the extract port by one column in the forward direction, and finally to 2/2/2/2 (3/4t_s–t_s) by shifting the raffinate port backward by one column. The configuration 2/1/2/3 explicates that there are two columns each in sections P and R, while one column in section Q and three columns in section S, respectively. As a result, in a four-sub-time interval Varicol® process, there are four different column configurations for the four subintervals due to local switching during one global switching period. The number of columns in each zone varies with time within a global switching period, but the number of columns in each zone returns to the starting value at the end of the global switching period. In terms of the time average number of columns per zone this corresponds to 2.5/1.25/1.75/2.5 for the above example. Therefore, locations of input/output ports

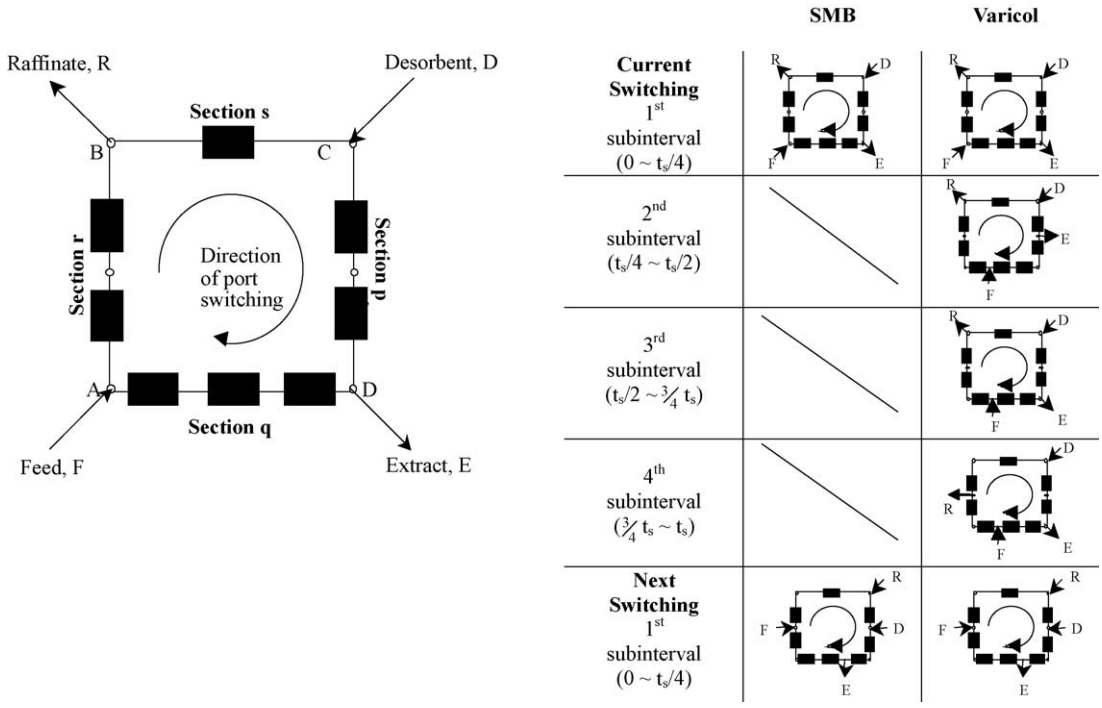

Fig. 2. (a) Schematic diagram of an eight-column SMBR system. (b) Principle of operation of the eight-column SMB and an equivalent four-subinterval Varicol (port switching schedule).

in a Varicol® process are quite different from that of the SMB process. Note that in principle it is possible that a port may shift more than once during one global switching period, either forward or even in backward direction. As a result, Varicol® process could be programmed with several possible column configurations, which endow more flexibility compared to a SMBR process without requiring any additional fixed cost. In the open literature, the reported results on the non-reactive Varicol® process are those for chiral separation [31,32] and glucose–fructose separation [27], and for reactive Varicol® for the synthesis of methyl acetate [35] and MTBE [36].

3. Mathematical model

The mathematical model together with the initial and boundary conditions that completely define the SMBR system for the inversion of sucrose were reported by Azevedo and Rodrigues [28], which was based on TMBR model. However, in this work in the optimization study the SMBR model is used, as it is more realistic representation of the experimental set-up. The governing differential mass balance equations and initial and boundary conditions are given in Appendix A. The details pertaining to the adsorption isotherms and the experimental set-up are summarized in Table 1. Sucrose is assumed to react in the inter-particle fluid phase and at the surface of the resin. This is accounted for by the term $(1 + \nu K_e)$, present in the Eqs. (A1) and (A4). The reaction products, viz., glucose and fructose are assumed to have linear isotherms [11,37,38]. The diffusion within the adsorbent particle is described by means of a bi-linear driving force approximation [39]. Sucrose adsorption on to the adsorbent particles was not considered in the model, since Azevedo and Rodrigues [5] experimentally observed that the adsorption constant of sucrose is nearly equal to the particle porosity. The diffusion of

Table 1
Experimental conditions used and measured model parameters [5]

Experimental set-up	
Parameter	Value
Number of columns, N_{col} (–)	12
Length of the column, L_{col} (cm)	29
Diameter of the column, d_{col} (cm)	2.6
Operating temperature, T (K)	328
pH (–)	4.5
Maximum allowable pressure, P (bar)	120

Experimentally measured model parameters (at $T = 328$ K)

Parameter	Glucose	Fructose
Pe (–)	1500	1500
Bi_m (–)	500	500
ε (–)	0.4	0.4
ε_p (–)	0.1	0.1
K (–)	0.17	0.43
$K' = K + \varepsilon_p$ (–)	0.27	0.53
$K' \frac{1-\varepsilon}{\varepsilon}$ (–)	0.405	0.795
$10^2\, k_p$ (s^{-1})	4.17	4.17
$10^2\, k_\mu$ (s^{-1})	2.17	2.17
$10^2\, k_h$ (s^{-1})	3.15	2.217
$10^2\, k_r$ (s^{-1})	83.87	83.87
K_e (–)	5.0	5.0

Table 2
Comparison of experimental and TMR model results reported by Azevedo and Rodrigues [5] with our SMBR model predicted results

Operating variables		Performance of the SMBR			
Parameter	Value	Parameter	Experimental value [5]	Predicted value[a]	Our prediction[b]
Q_p (ml/min)	35.38	Pur_F (%)	90.0	92.6	90.7
Q_q (ml/min)	26.27	Pur_G (%)	96.3	96.1	96.5
Q_r (ml/min)	29.89	PR_F (kg/m^3 h)	7.78	7.55	7.65
Q_s (ml/min)	24.0	PR_G (kg/m^3 h)	7.07	7.40	7.20
Q_F (ml/min)	3.62	PR_{Enz} (kg/g)	0.102	0.102	0.102
Q_R (ml/min)	5.89	$C_{F,E}$ (g/l)	15.78	15.3	15.52
Q_D (ml/min)	11.38	$C_{G,R}$ (g/l)	22.18	23.2	22.59
Q_E (ml/min)	9.11	X_S (%)	100	Not reported	100
t_s (in)	3.4				
C_{SF} (g/l)	80.0				
C_{ED} (g/l)	0.25				

[a] Based on TMBR model [5].
[b] Based on SMBR model (this work).

sucrose into the adsorbent was also not considered since the reaction rate constant is much larger than the pore diffusion characteristic time [5].

The PDEs (governing equations given in Appendix A) were discretized in space using the finite difference method to convert them into a system of coupled ODE-IVPs (method of lines). These stiff IVP-ODEs were solved using the subroutine, DIVPAG (which is based on Gear's method), in the IMSL library. Since periodic switching is imposed on the system, the reactor-separator works under transient conditions. However, a cyclic (periodic) steady state with a period equal to the global switching time is eventually reached after several switching. For both SMBR and Varicol® process, the periodic steady state was always attained after about 10 switching cycles around the unit, as was reported in the work of Azevedo and Rodrigues [5]. The simulated values predicted by the above mathematical model matched very well with the values reported by Azevedo and Rodrigues [5]. The SMBR performance was evaluated based upon the productivity and purity achieved at both the extract and the raffinate ports. These quantities were defined in the work of Azevedo and Rodrigues [5,28] as:

$$PR_F = \text{kg of fructose at extract port/m}^3 \text{ adsorbent/h} \quad (4)$$

$$PR_G = \text{kg of glucose at raffinate port/m}^3 \text{ adsorbent/h} \quad (5)$$

$$Pur_F = \text{kg of fructose/kg of fructose and glucose at extract port} \quad (6)$$

$$Pur_G = \text{kg of glucose/kg of fructose and glucose at raffinate port} \quad (7)$$

$$X_S = \text{kg of sucrose reacted/kg of sucrose fed} \quad (8)$$

Table 2 compares the simulation results obtained from the current study with those reported in the literature for a particular experimental run [5]. It is worth noting that all the values reported in the literature were all based on the TMBR model, while the results from the present work are all based on the SMBR model. Moreover, prediction of the experimental results with SMBR model is much better than that with the TMBR model.

4. Optimization of the SMBR and Varicol® systems

The experimental conditions used by Azevedo and Rodrigues [5] were selected from within the SMB operation triangle based on equilibrium theory. It should be noted that the equilibrium theory is only applicable for pure separation only and not yet has been developed for reactive separation. Moreover, the triangle for complete separation would shrink when there are mass transfer resistances. They also performed optimization studies using an optimization algorithm that is an extension of that developed by Biressi et al. [40]. However, the optimization technique was for single objective function optimization. The purity and productivity of the two different products as a result of the inversion of sucrose, i.e., fructose and glucose, were found to change in conflicting ways with changes in the operating parameters. In real life problems such as the SMBR system considered in this work, there is often more than one objective function, which are equally significant and need to be considered simultaneously. In such cases, a more thorough multi-objective optimization study [41,42] has to be done so that the design of the SMBR set-up could be more accurate, comprehensive and economical. Throughout this study the focus is given more on maximizing the productivity of fructose while minimizing the water consumption, which is as discussed earlier the major concern for most of the sugar industries.

Two distinct types of problems may be considered in the multi-objective optimization of the performance of reactive SMB (and Varicol®) systems. One is the performance enhancement of the existing set-up by suitably determining the optimal operating parameters and running the plant at those conditions. The other one is the optimization at the design stage (the design of a new SMBR unit) for efficiently handling sucrose inversion by enzymatic action and simultaneous separation of fructose–glucose. The design strategy for the

SMBR (and Varicol®) proposed in this work consists of determining the optimal geometric parameters (such as length, number and sequence of columns) and the operating conditions (flow rates, switching time, etc.) that allow a desired substrate conversion and purity at the outlet streams with the maximum productivity using minimum desorbent while at the same time without exceeding pressure drop limits imposed by the packing material. A few two-objective functions optimization has been reported in this work which fall under both the categories of the optimization problems mentioned above. One can, of course, consider more than two objective functions but analysis of the results become cumbersome as one has to deal with Pareto surfaces. It is to be emphasized that there is no end to the variety of multi-objective optimization problems, which could be formulated and studied, and in this article, only a few simple examples have been dealt with, to illustrate the concepts, techniques and interpretation of results.

The state-of-the-art evolutionary optimization technique based on genetic algorithm [14] is used in this study. The algorithm mimics the process of natural selection and natural genetics. The Darwinian principle of 'survival of the fittest' is used to generate improved solutions [14]. The sorting and sharing mechanism introduced in elitist non-dominated sorting genetic algorithm, NSGA-II [14], has paved the way for multi-objective optimization. The recent modification with jumping genes [26] has improved the diversity of hypothetic mating pool leading to a much better spreading of solution at increased convergence speed. The jumping gene operations adapt a modified mutation operator, borrowing from the concept of jumping genes in natural genetics. McKlintock [43] suggested in the 1940s that jumping genes are DNAs that could jump in and out of chromosomes and can generate genetic diversity in natural populations, which is exploited in NSGA-II-JG. Details of methodology and applications of different adaptations of NSGA in chemical engineering can be obtained elsewhere [26]. However, a short discussion has been added in Appendix B. In all optimization runs presented in this work, 50 chromosomes (solutions) were considered and results are presented after 50 generations. The CPU time taken to generate one Pareto set is about 11 min on the CRAY J916 supercomputer.

Case 1 *(Performance enhancement of the existing set-up).* Azevedo and Rodrigues [5] reported single objective function optimization results for maximization of enzyme productivity subject to conversion greater than 99%, and purity of both extract and raffinate streams greater than 95% as constraints. However, the single objective optimization of SMBR (and Varicol®) operation is less meaningful due to complex interplay of relatively large number of parameters. The primary objective of inversion of sugar plant, for example in soft drinks production plant, is to maximize production of 60% concentrated fructose using minimum solvent. Water consumption is usually very large (ca. 50 l of water is required per kg of fructose product) and therefore, reduction of water consumption is one of the primary objectives in industry. Thus, the first multi-objective optimization problem solved is the maximization of fructose productivity using minimum solvent (water consumption) for an existing plant. The formulation can be represented mathematically as:

$$\text{Maximize} \quad I_1 = \text{PR}_\text{F}[t_\text{s}, Q_\text{R}, Q_\text{D}] \tag{9a}$$

$$\text{Minimize} \quad I_2 = Q_\text{D}[t_\text{s}, Q_\text{R}, Q_\text{D}] \tag{9b}$$

$$\text{Subject to} \quad \text{Pur}_\text{F} \text{ and } \text{Pur}_\text{G} \geq 60\% \tag{9c}$$

$$\text{PR}_\text{F} \text{ and } \text{PR}_\text{G} \geq 7.0\,\text{kg/m}^3 \text{ solid/h} \tag{9d}$$

The choice of two objectives enable the production of fructose using minimum water subject to target purities of extract and raffinate streams greater than 60% and productivities greater than the experimental reported values [5]. Three decision variables were used for this optimization study as indicated in Eq. (9): switching time, t_s, raffinate stream flow rate, Q_R, and desorbent flow rate, Q_D. In order to be able to compare our results with those of an existing set-up [5], we fixed (see Table 3) the values of length, diameter and number of columns in each sections, feed flow rate and flow rate in section S, concentration of sucrose in feed and enzyme in desorbent and temperature corresponding to their experimental values. Since only four flow rates could be selected independently, while the other four are determined by mass balance equations around points A–D (see Fig. 1), the remaining two flow rates (in this case, Q_D and Q_R) were used as decision variables. The third decision variable is the switching time t_s,

Table 3
Description of the multi-objective optimization problems solved for the SMBR systems

Problem no.	Objective function	Decision variables	Constraints	Fixed parameters
1 (Existing)		$120 \leq t_\text{s} \leq 360$ s $2.4 \leq 10^4\, Q_\text{R} \leq 4.2$ m^3/h $3 \leq 10^4\, Q_\text{D} \leq 9$ m^3/h	$\text{PR}_\text{F} \geq 7.0$ kg/m^3 solid/h; $\text{PR}_\text{G} \geq 7.0$ kg/m^3 solid/h; $\text{Pur}_\text{F} \geq 60\%$; $\text{Pur}_\text{G} \geq 60\%$	$L_\text{col} = 0.29$ m, $d_\text{col} = 0.026$ m, $T = 323$ K $N_\text{col} = 12$, $Q_\text{F} = 2.172 \times 10^{-4}$ m^3/h $Q_\text{S} = 1.44 \times 10^{-3}$ m^3/h, p/q/r/s = 3/2/5/2 $C_\text{F} = 8 \times 10^4$ kg/m^3, $C_\text{E} = 250$ kg/m^3
2 (Design)	Max PR_F; min Q_D	$30 \leq t_\text{s} \leq 240$ s $0.6 \leq 10^4\, Q_\text{R} \leq 3$ m^3/h $0.3 \leq 10^3\, Q_\text{D} \leq 1.2$ m^3/h $0.1 \leq L_\text{col} \leq 0.3$ m $1 \leq$ p, q, r ≤ 5		Same as problem 1 except $N_\text{col} = 8$, 10 or 12, and p, q, r and L_col are decision variables

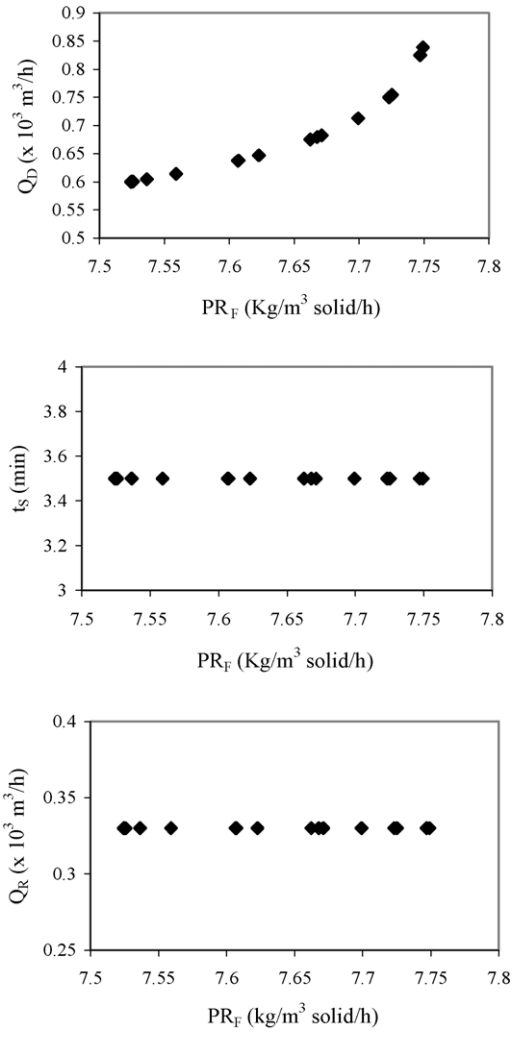

Fig. 3. Pareto-optimal solutions and corresponding the decision variables for the 12-column SMBR system (problem 1).

which clearly has a strong influence on the purity of the outlet streams. The bounds for t_s lie between the breakthrough times of the two components for the resin used as adsorbent.

Fig. 3 shows the Pareto-optimal solution and the decision variables corresponding to each of the points on the Pareto for the optimization problem formulated in case 1. The figure clearly shows that the points do, indeed, constitute a Pareto set, i.e., as the productivity of fructose increases (desirable), it requires more solvent (undesirable). The optimum values of switching time and raffinate flow rate obtained are 3.5 min and 5.5 ml/min respectively. Although, constraint on purity was only 60%, the purity of fructose achieved was 90%, same as in the experimental study. The conversion of sucrose achieved was 100%. The optimal values of fructose productivity obtained were observed to be only slightly better (about 0.3%) since only three decision variables were used, and therefore, restricting the scope for improvement considerably. However, the optimization results in range of optimal PR_F for a range of Q_D.

Case 2 *(Optimization at the design level: optimal column length and configuration)*. Optimization at the design stage provides far more freedom than when one is constrained to optimize the performance with an existing set-up. At the design stage, several additional decision variables become available for optimization. A meaningful optimization problem would be same as in case 1 except additional decision variables as length, number and distribution of columns in different sections p, q, r and s. The cost of adsorbent is always one of the key deciding factors for the implementation of the SMB units. The existing set-up considered in problem 1 consisted of 12 columns each 0.29 m long. In this case, optimization problem was formulated to determine optimal column length and distribution of columns in different sections for total number of columns, N_{col}, equal to 8, 10 and 12. The complete optimization formulation is given in Table 3. The Pareto sets for this problem is shown in Fig. 4. When compared with problem 1, the performance of reactor is seen to have considerably improved. For equivalent 12-column SMBR system and for the same desorbent consumption of 10 ml/min, the productivity has dramatically increased from 7.52 to 17.46 kg/m^3 solid/h. This was achieved primarily by optimizing column length and by optimally distributing the

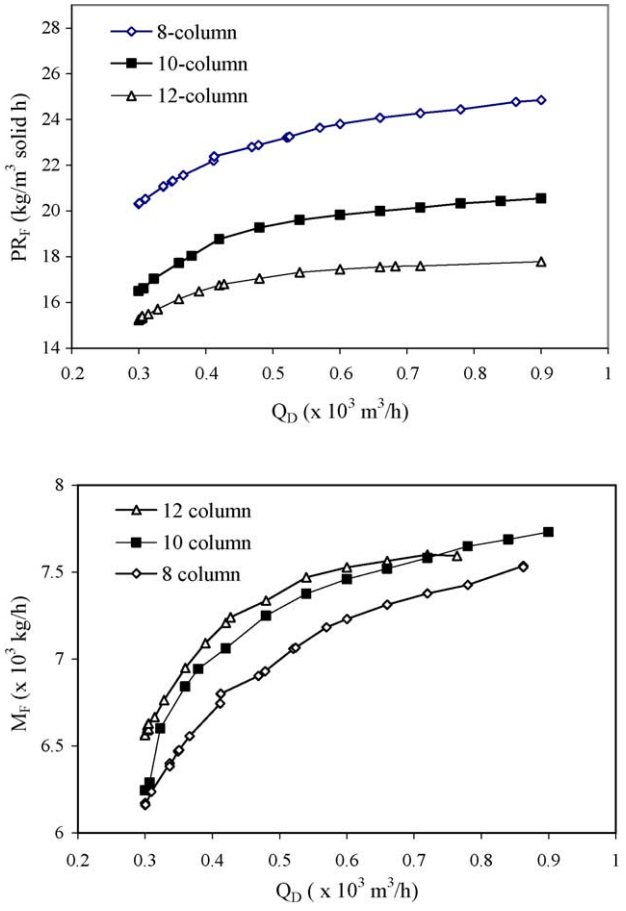

Fig. 4. Comparison of PR_F and M_F between 8-, 10- and 12-column SMBR system.

columns in different sections. The optimum column length and configuration (distribution) obtained for the design stage are 0.113 m and χ = 3/1/4/4 compared to 0.29 m and 5/2/3/2 used for the existing set-up. Note that although in the optimization formulation we have used only two objectives (PR_F and Q_D), but in reality it involves three objectives (the third one being minimization of total adsorbent volume) due to the manner in which productivity (amount of fructose produced in kg/h per unit volume of total adsorbent used) is defined. Obviously, the optimum amount of glucose produced (Q_R) in the raffinate stream is reduced from 3.3×10^{-4} to 1.29×10^{-4} m^3/h due to the increased production of fructose at the extract port. The optimum switching time has reduced to 1.51 min (from 3.5 min) as the column length is reduced. Even though the actual productivity (in kg/m^3 solid/h) is increased but the mass flow rate of fructose (M_F, kg/h) has actually decreased compared to problem 1. However, the reduction in M_F is only 9.5% compared to total reduction in V_{solid} by 61% in the case of design optimization compared to the existing set-up for the 12-column SMBR system. Hence, the present optimization formulation helps to design the SMBR system for efficient utilization of adsorbent in producing fructose using minimum solvent.

In order to determine whether 12 columns are necessary, optimization was carried out for 10 and 8 total columns. The Pareto-optimal solution for the 8- and 10-column SMBR unit are also shown in Fig. 4. The Pareto set of Q_D and PR_F followed the similar trend line as in the earlier cases, but yielding even higher values of fructose productivity at a particular desorbent flow rate. For example, for a given Q_D of 6×10^{-4} m^3/h, an improvement in PR_F from 17.46 (for a 12-column SMBR) to 19.84 (for a 10-column SMBR) to 23.8 kg/m^3 solid/h (for an eight-column SMBR) was possible. The optimum column length, configuration, switching time and raffinate flow rate for all the points in the Pareto set for a particular set-up was found to be constant and are respectively, 0.113 m, χ = 4/4/3/1, t_s = 1.51 min, $Q_R = 1.29 \times 10^{-4}$ m^3/h for 12-column; 0.118 m, χ = 4/1/2/3, t_s = 1.48 min, $Q_R = 1.46 \times 10^{-4}$ m^3/h for 10-column; and 0.119 cm, χ = 2/2/2/2, t_s = 1.51 min, $Q_R = 1.42 \times 10^{-4}$ m^3/h for eight-column SMBR system. Once again, even though PR_F increased when N_{col} is reduced from 12 to 8 columns, the values of M_F also decreased. This is also shown in Fig. 4. It clearly shows that by adding two columns to an eight-column system, the improvement is considerable but further addition of two columns to make it a 12-column system, the increase is not significant. For example, for Q_D of 6×10^{-4} m^3/h, when performance is compared relative to 12-column SMBR system at the design stage optimization, the reduction in M_F is only 1.12% compared to total reduction in V_{solid} by 13% for 10-column system, while reduction in M_F of 30% for reduction in V_{solid} of 4.3% for eight-column SMBR system. Design stage optimization leads to production of 2.91×10^4 kg of concentrated fructose per m^3 of solid adsorbent per m^3 of water consumption compared 1.25×10^4 kg fructose/m^3 solid/m^3 water for a 12-column SMBR set-up. The efficiency increases to 3.97×10^4 kg fructose/m^3 solid/m^3 water for an eight-column SMBR set-up.

Case 3 *(Modification of SMBR: variable feed flow rates).* One of the limitations of the SMB (and SMBR) system is that during much of the operation, the stationary phase in some of the columns are either completely free of solutes, or contains only product so that the separation capacity is significantly reduced, thereby the overall efficiency of the system is low. Most SMB studies in the open literature have used constant flow rates during each switching period. However, it is generally observed that the concentration profile of a typical SMB process shows a maximum value at the beginning of the switch, slowly decreases thereafter, and finally decreases very rapidly near the end of the switching period. In such a case, one could consider of changing the various internal flow rates depending on the process requirement to increase productivity or purity of the desired product. This can be achieved by the use of a non-synchronous switching as in Varicol® [31,32], which is discussed earlier and is considered later. Alternatively, one could also contemplate in improving the efficiency by using variable feed (or desorbent) flow rates during a global switching period to either increase throughput or productivity [42]. One may consider of introducing more feed at the beginning of the cycle and collecting more extract or raffinate depending on the requirement of the process, and slowly decreasing the feed flow rate towards the end of the switching period. However, such a change in flow rates would affect concentration profiles and may even lead to decreased productivity unless a rigorous optimization study is performed taking into consideration complex interplay of all the process parameters (Table 4).

In order to evaluate the efficacy of this approach, and to determine the extent to which the performance of SMBR could be improved by using variable feed flow rate, three different optimization problems were formulated as described in Table 5. In all the formulations, maximization of mass flow rate of fructose (M_F, g/h) is used as one of the objective functions instead of maximization of productivity

Table 4
Comparison of optimization results for 8-, 10- and 12-column SMBR systems

	Existing	Design stage		
N_{col}	12	12	10	8
L_{col}, m	0.29	0.113	0.118	0.119
10^4 V_{col}, m^3	18.48	7.2	6.26	5.05
10^4 Q_D, m^3/h	6	6	6	6
PR_F, kg/m^3 h	7.52	17.46	19.84	23.81
10^2 M_F, kg/h	1.39	1.26	1.24	1.2
10^{-4} Efficiency[a]	1.25	2.91	3.31	3.97

[a] Efficiency is defined as productivity of fructose (kg/m^3 h—solid used) per unit eluent (water) consumption (m^3/h).

Table 5
Description of the multi-objective optimization problems solved for the modified SMBR

Problem no.	Objective function	Decision variables	Constraints	Fixed parameters
3 SMBR (fixed feed)		$9 \le 10^5\, Q_R \le 30\ m^3/h$ $2.4 \le 10^4\, Q_D \le 12\ m^3/h$ $30 \le t_s \le 240\ s$ $0.1 \le L_{col} \le 0.3\ m$ $1 \le p, q, r \le 5$		$N_{col} = 8$, $d_{col} = 0.026\ m$, $T = 323\ K$ $Q_F = 2.172 \times 10^{-4}\ m^3/h$ $C_F = 8 \times 10^4\ kg/m^3$, $C_E = 250\ kg/m^3$ $Q_S = 1.44 \times 10^{-3}\ m^3/h$
3a SMBR (discrete feed)	Max M_F; min Q_D	$9 \le 10^5\, Q_R \le 30\ m^3/h$ $2.4 \le 10^4\, Q_D \le 12\ m^3/h$ $1.8 \le 10^4\, (Q_{F1}, Q_{F2}, Q_{F3}) \le 3\ m^3/h$	$Pur_F \ge 60\%$; $Pur_G \ge 60\%$	Same as problem 3 and $t_s = 154.8\ s$, $L_{col} = 0.20\ m$, $\chi = 4/1/1/2$
3b SMBR (continuous feed)		$9 \le 10^5\, Q_R \le 30\ m^3/h$ $2.4 \le 10^4\, Q_D \le 12\ m^3/h$ $0.5 \le b, c \le 5.0$		Same as problem 3a
4 Varicol		$9 \le 10^5\, Q_R \le 30\ m^3/h$ $2.4 \le 10^4\, Q_D \le 12\ m^3/h$ $30 \le t_s \le 240\ s$ $0.1 \le L_{col} \le 0.3\ m$ χ (32 possible combinations)		Same as problem 3

as used in problems 1 and 2. In the first case (problem 3 in Table 5), optimal Pareto solutions are obtained at the design stage for an eight-column SMBR system in which feed flow rate was maintained constant at $2.172 \times 10^{-4}\ m^3/h$. Subsequently, two more optimization problems were solved with variable (distributed) feed flow rate. In problem 3a, four discrete sub-feed intervals were used while in problem 3b, feed flow rate was allowed to change continuously. In problem 3a, the feed flow rate was not kept constant at $Q_F = 2.172 \times 10^{-4}\ m^3/h$ for the entire switching interval, instead was allowed to vary according to the following equations:

$$1.8 \times 10^{-4} \le Q_{F1}, Q_{F2}, Q_{F3} \le 3 \times 10^{-4}\ m^3/h \quad (10a)$$

$$Q_{F4} = 4Q_F - (Q_{F1} + Q_{F2} + Q_{F3}) \quad (10b)$$

Eq. (10b) is used to ensure that total feed flow rate is same as that of the constant feed flow case (problem 3), and therefore, the optimum results can be compared. In problem 3b, the feed flow rate was allowed to vary continuously according to:

$$Q_F = a - bt - ct^2 \quad (11)$$

where a, b and c are constants and t is the time such that $0 \le t \le t_s$. The constants b and c were chosen as decision variables while a was calculated by integrating the above equation over the integral 0 to t_s Eq. (12), so as to ensure again that the total feed remains constant (3.62 ml/min):

$$a = Q_F + \frac{b}{2}t_s + \frac{c}{3}t_s^2 \quad (12)$$

In the later two cases, length of each column (L_{col}), switching time (t_s) and column configuration (χ) were kept fixed at the optimum values for problem 3. The detailed optimization formulation is described in Table 5. Fig. 5 compares the Pareto-optimal solutions for the three cases. The results show that the significant improvement is possible when one uses variable feed flow rate instead of fixed feed flow rate and further improvement is possible if one uses continuous variation in contrast to discrete variation of the feed flow rate. The

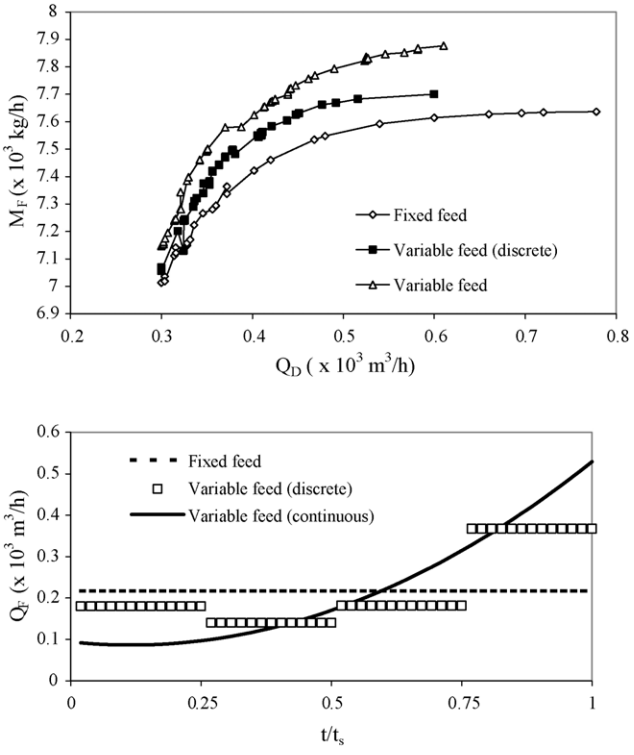

Fig. 5. Comparison of eight-column SMBR system with fixed feed, variable feed (discrete and continuous). The profile for Q_F is for $Q_D = 4.21 \times 10^{-4}\ m^3/h$.

optimum flow rate of glucose at the raffinate port was found out to be 1.26×10^{-4} m^3/h for all three cases. Typical optimal feed flow rate for different cases within a switching period at steady state is shown in Fig. 5b for $Q_D = 4.2 \times 10^{-4}$ m^3/h. The figure shows that the optimal trend is for lower Q_F at the beginning of a cycle and gradual increase in Q_F as time progresses when continuous variation was allowed. Almost similar trend was observed when Q_F was allowed to vary in four discrete steps. At the beginning of any switching period, feed rate required is low due to recycle from section S to P and it gradually increases towards the end of the switching period. The improvement in the performance achieved can be easily explained by recalling the earlier discussion about the possibility of performance improvement by deliberately increasing (or decreasing) the flow rates. Due to the special purity requirements (in this case 60%), probably during the start of the switching period the solids are saturated, and therefore can handle lesser feed, while due to the flushing away of the solids during the switching period the feed requirement increases. For fixed values of Q_R and Q_D, Q_E would increase when Q_F is increased, thereby increase the amount of fructose collected. In other words, the system tends to operate such that higher quantity of fructose is collected when the concentration of fructose is higher while when the concentration is lower lesser quantities are collected, thus keeping the mean flow rates over the switching time constant. Note that the productivity is not just a mean of the intermediate values, but is the integral of the product of flow rate and concentration over the entire switching period.

Case 4 *(Modification of SMBR: Varicol® system)*. It has been reported as discussed earlier that Varicol system can perform better than the traditional SMB systems due to the flexibility in its operation. It could aid in reducing the volume of adsorbent required, or for a fixed total adsorbent, it can increase throughput or productivity. Hence, optimization study was performed for a four sub-time interval eight-column Varicol® system to determine the extent of improvement that can be obtained over an equivalent SMBR system. With eight total columns in the Varicol unit, there could be a large number of possible column configurations (χ). The mathematical formulation of the optimization formulation is given in Table 5 (problem 4).

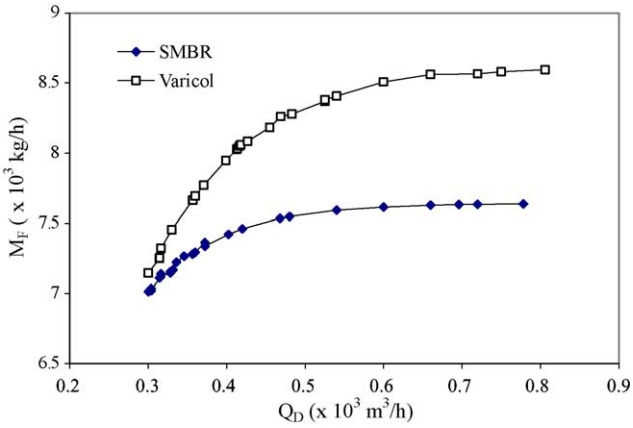

Fig. 6. Comparison of optimal performance between an eight-column Varicol system with eight-column SMBR system.

The Pareto-optimal solution of the eight-column Varicol® system and the corresponding decision variables are illustrated in Fig. 6. The optimal Q_D–M_F trend line obtained was similar to problem 3 (Fig. 5). Fig. 6 clearly shows that mass flow rate of fructose produced per unit time is significantly higher for a fixed eluent flow rate, or same amount of fructose can be produced using less eluent. This is possible in Varicol® process due to non-synchronous switching compared to synchronous switching in SMBR system. However, it was observed that the length of the column chosen is higher (0.26 m as compared to 0.20 m for SMBR). The purpose of still keeping L_{col} as one of the decision variables was to see if an optimal L_{col} could be obtained for the Varicol®. Three different optimal column configurations for the four subinterval eight-column Varicol® system were obtained. Most part of the curve was dominated by the configuration (χ) 4/1/2/1, 4/1/2/1, 3/1/3/1, 3/1/3/1 except by the configuration 4/1/2/1, 3/2/2/1, 3/1/3/1, 3/1/3/1 for Q_D between 4.8×10^{-4} to 5.4×10^{-4} m^3/h. The trend clearly shows that more columns are needed in section P in the beginning of the subinterval while more are needed in section R near the end of the cycle. This probably is due to the fact that fructose is more strongly adsorbed than glucose and tends to diffuse very slowly. Sections P and R being the zones for desorption and adsorption for the more strongly adsorbed compound, respectively, more solids (adsorbent) are required in these sections when the objective is to increase the productivity of fructose

Table 6
Comparison of optimal results between SMBR with fixed and distributed (discrete and continuous) feed and Varicol process for eight-column systems (see Figs. 5 and 6)

	SMBR with feed flow rate			Varicol
	Fixed	Variable (discrete)	Variable (continuous)	Non-synchronous switching
N_{col}	8	8	8	8
L_{col}, m	0.20	0.20	0.20	0.265
$10^4 \times V_{col}$, m^3	8.52	8.52	8.52	11.26
$10^4 \times Q_D$, m^3/h	6	6	6	6
$10^3 \times M_F$, kg/h	7.61	7.70	7.87	8.51
kg-F/m^3-water	12.69	12.83	13.12	14.18

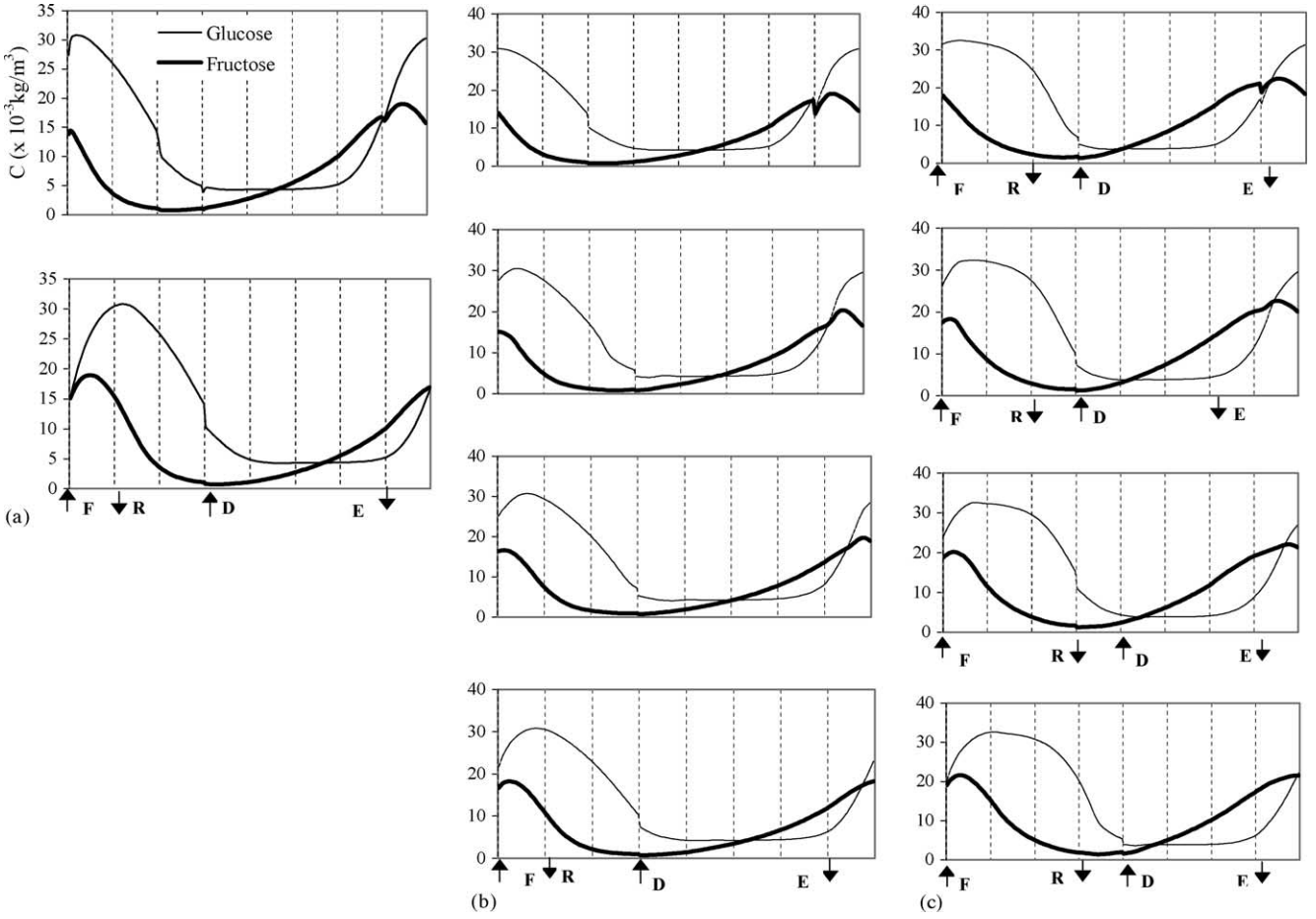

Fig. 7. Concentration profiles for eight-column SMBR (a), distributed feed (b) and Varicol (c) for the point, $Q_D = 0.48 \times 10^{-3}$ m^3/h.

using less eluent. Fig. 7 elucidates the concentration profiles for glucose and fructose for the eight-column SMBR with fixed and variable (discrete) feed and the four-subinterval Varicol® system corresponding to the optimal point with desorbent flow rate equal to 4.8×10^{-4} m^3/h. Table 6 compares the optimal performance of eight-column SMBR system with fixed and distributed feed with eight-column Varicol system for a specified Q_D of 6×10^{-4} m^3/h. The table shows that the performance in terms of optimal mass flow rate for fructose can be increased from 7.61×10^{-3} kg/h for fixed feed to 7.70×10^{-3} kg/h for discrete variable feed flow rate to 7.87×10^{-3} kg/h for continuous variable feed flow rate to 8.51×10^{-3} kg/h for Varicol® system. The efficiency in terms of fructose produced (in kg/h) per unit water consumption (in m^3) increased from 12.69 to 14.18.

5. Conclusions

Continuous large-scale separations using simulated moving bed (SMB) technology have received great interest in recent years. In this work, systematic multi-objective optimization studies for the inversion of sucrose to produce high fructose syrup are considered. Optimal operating conditions for both an existing system as well as at the design stage were determined for maximum production of at least 60% concentrated fructose (used in soft drinks) while using minimum solvent. Several decision variables were used that include length and distribution of columns in different sections in addition to switching time and various flow rates. Effect of two modifications of traditional SMB, namely distributed feed and non-synchronous switching (Varicol® process) were studied to determine the extent of performance improvement compared to the SMBR system. The optimization was performed using a new state-of-the-art AI-based non-traditional but robust optimization technique based on genetic algorithm, non-dominated sorting genetic algorithm with jumping genes (NSGA-II-JG). Pareto-optimal solutions were obtained in all cases and the results show that significant improvement is possible, particularly for distributed feed and Varicol® operation.

Appendix A

Differential mass balances, global, and in solid phase for species *i* in column *k*:

For glucose and fructose:

$$\frac{\partial C_{i,k}}{\partial \theta} + \upsilon \frac{\partial \bar{q}_{i,k}}{\partial \theta} = \frac{\psi_k}{Pe_k}\frac{\partial^2 C_{i,k}}{\partial \chi^2} - \psi_k \frac{\partial C_{i,k}}{\partial \chi} + k_t t_s (1 + \nu K_e)\left(\frac{\sigma_i R_j}{k_r}\right) \quad \text{(A1)}$$

$$\frac{\partial \bar{q}_{i,k}}{\partial \theta} = \alpha_i \frac{\mathrm{Bi}_{m_k}}{5 + \mathrm{Bi}_{m_k}}(\bar{q}^*_{i,k} - \bar{q}_{i,k}) \quad \text{(A2)}$$

$$\bar{q}^*_{i,k} = K_i' C_{i,k} \quad \text{(A4)}$$

For sucrose:

$$\frac{\partial C_{i,k}}{\partial \theta} = \frac{\psi_k}{Pe_k}\frac{\partial^2 C_{i,k}}{\partial \chi^2} - \psi_k \frac{\partial C_{i,k}}{\partial \chi} + k_t t_s (1 + \nu K_e) \times \left(\frac{\sigma_i R_j}{k_r}\right) \quad \text{(A5)}$$

where $\sigma_I = 0.526$ for glucose and fructose, while $\sigma_I = -1$ for sucrose.

$$R_j = k_r \left(\frac{C_{\mathrm{Suc},j} \times C_{\mathrm{Enz},j}}{K_{mm} + C_{\mathrm{Suc},j}}\right) \quad \text{(A6)}$$

For the enzyme, invertase:

$$\frac{\partial C_{i,k}}{\partial \theta} = \frac{\psi_k}{Pe_k}\frac{\partial^2 C_{i,k}}{\partial \chi^2} - \psi_k \frac{\partial C_{i,k}}{\partial \chi} \quad \text{(A7)}$$

where $\chi = z/L_{col}$ and $\theta = t/t_s$.

The boundary conditions are:

$$C^{in}_{i,\chi} = C_{i,k}(0, \theta) - \frac{1}{Pe_k}\frac{\partial C_{i,k}}{\partial \chi} \quad \text{(A8)}$$

$$\frac{\partial C_{i,k}}{\partial \chi}(1, \theta) = 0 \quad \text{(A9)}$$

The initial conditions are:

$$C_{i,k}(\chi, 0) = C^0_{i,k}(\chi) \text{ and } \bar{q}_{i,k}(\chi, 0) = \bar{q}^0_{i,k}(\chi) \quad \text{(A10)}$$

The dimensionless parameters used in the above equations are:

$$\psi_k = \frac{U_{F_z} t_s}{L_{col}} \text{ and } \alpha_i = k_{h_z} t_s \quad \text{(A11)}$$

where

$$k_{h_z} = \left(\frac{K + \varepsilon_p}{k_p} + \frac{K}{k_\mu (K + \varepsilon_p)}\right)^{-1} \quad \text{(A12)}$$

$$K' = K + \varepsilon_p,\ Pe_k = \frac{U_{F_z} L_{col}}{D_{ax_k}} \quad \text{(A13)}$$

and

$$\mathrm{Bi}_{m_k} = \frac{k_{f_k} R_p}{D_{pe}} \quad \text{(A14)}$$

The mass balance at the node is then, $C^{in}_{i,k} = C_{i,k-1}(1, \theta)$, except if the column follows feed or desorbent port. In that case:

$$C^{in}_{i,k} = \frac{Q_F C_{i,F} + Q_S C_{i,k-1}(1, \theta)}{Q_P} \quad \text{and}$$

$$C^{in}_{i,k} = \frac{Q_Q C_{i,k-1}(1, \theta)}{Q_R}, \text{ respectively.} \quad \text{(A15)}$$

Appendix B. A note on the optimization technique based on genetic algorithm (GA)

GA is a search technique developed by Holland [44] that mimics the process of natural selection and natural genetics. In this algorithm, a set of decision variables are first coded in the form of a set of randomly generated chromosomes, thereby creating a 'population (gene pool)'. A model of the process is then used to provide values of the objective function (reflects its 'fitness' value) for each chromosome. The Darwinian principle of 'survival of the fittest' is used to generate a new and improved gene pool (new generation). This is done by preparing a 'mating pool', comprising of copies of chromosomes, the number of copies of any chromosome being proportional to its fitness (Darwin's principle). Pairs of chromosomes are then selected randomly, and pairs of daughter chromosomes generated using three operations (reproduction, crossover and mutation) similar to those in genetic reproduction. The gene pool evolves, with the fitness improving over the generations.

In order to handle multiple objective functions and to find Pareto-optimal solutions, the simple genetic algorithm (SGA) has been modified to non-dominated sorting genetic algorithm I (NSGA-I), which differs from SGA only in the way the selection operator works [14]. NSGA-I uses a ranking selection method to emphasize the good points and a niche method to create diversity in the population without losing a stable sub-population of good points. In the new procedure, several groups of non-dominated chromosomes from among all the members of the population at any generation are identified. To distribute (spread out) the points evenly, the fitness value is assigned according to a sharing procedure. The population is found to converge rapidly to the Pareto set.

However, experience with NSGA-I indicates that this algorithm has some disadvantages. The sharing function used to evaluate niche count of any chromosome requires the values of two parameters, which are difficult to assign a priori. In addition, NSGA-I does not use any elite-preserving operator and so, good parents may get lost. Deb [14] has recently developed an elitist non-dominated sorting genetic algorithm (NSGA-II) to overcome these limitations. In NSGA-II, a different sorting and sharing method is used, which reduces the numerical complexity to MN_p^2 operations in contrast to MN_p^3 operations required for NSGA-I, where M is the number of

objective functions, and N_p is the number of chromosomes in the population.

Kasat et al. [45,46] recently introduced a modified mutation operator, borrowing from the concept of jumping genes (JG) in natural genetics. This algorithm is being called as NSGA-II-JG. This is a macro-macro mutation, and counteracts the decrease in the diversity created by elitism. The jumping genes operation is carried out after crossover and normal mutation in NSGA-II. A part of the binary strings in the selected chromosomes is replaced with a newly (randomly) generated string of the same length. Only a single jumping gene was assumed to replace part of any selected chromosome. This helps save considerable amounts of the computation time (at times, gives correct solutions, which are missed by other algorithms) and is important for compute-intense multi-objective problems like that of the SMB and Varicol process. Nandasana et al. [26] has reviewed recently the applications of different adaptations of NSGA in chemical engineering.

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